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Deformed scalar quantum electrodynamics as a phenomenological model for composite scalar particles

C I Ribeiro-Silva, E M F Curado and M A Rego-Monteiro

Centro Brasileiro de Pesquisas Físicas, Rua Xavier Sigaud 150, 22290-180, Rio de Janeiro, RJ, Brazil

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Abstract

We have constructed deformed scalar quantum electrodynamics where the scalar bosons are created and/or annihilated by the step operators of a generalized Heisenberg algebra and the photons are described in a standard way. One parameter, η, was introduced in those terms of the interaction Hamiltonian which have derivatives. We have computed the scattering process q-boson + photon $\rightarrow$ q-boson + photon up to second order in the coupling constant. We have found that the parameter, η, introduced is essential to preserve Lorentz and gauge invariance at the quantum level. We compare the cross-section for the scattering $2\gamma \rightarrow q$-boson $+q$-boson with the experimental data for $2\gamma \rightarrow \pi^+ + \pi^-$, where $\pi^\pm$ are the charged pions, obtaining good agreement in the region 0.55–0.7 GeV.

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1. Introduction

Quantum algebras first appeared in the investigation of integrable models and the Yang–Baxter equation by Kulish and Reshetikin [1]. Later, they were independently formalized by V G Drinfel'd [2] and M Jimbo [3] in their studies on the Yang–Baxter equation. These algebras are a generalization of the concept of symmetries and in the past two decades there has been much interest in understanding their physical properties and possible consequences of these structures in physics (see, for instance, [4–13]).

Among these quantum algebras there is an interesting class of algebras named deformed Heisenberg algebras [14–16]. They are deformations of the Heisenberg algebra by means of the introduction of one or more parameters. There has been much work studying these structures and properties. An interpretation of deformed Heisenberg algebras as describing phenomenologically composite particles has been explored in the literature in the last 15 years; see, for example, [17–24]. The basic argument is that the algebra of the creation and annihilation operators representing composite particles (or the step operators of general excitations of many point-like particles) becomes different from the usual Heisenberg algebra due to the internal degrees of freedom of their corresponding point-like particles; see a more detailed discussion, of this point, for example, in [20].

In order to explore these ideas, a program to construct a phenomenological quantum field theory (QFT) based on a generalized Heisenberg algebra (GHA) [25] was started [26, 27]. The purpose of this program is to give a phenomenological description of the interaction of composite particles. In [26, 27] a deformed scalar QFT was constructed and in [28] it was shown that this deformed model is renormalized up to second order in the coupling constant. Note that the deformed QFT constructed in the last three references obeys the important requirements of a consistent QFT such as Lorentz invariance, locality and renormalization up to second order in the coupling constant.

In this paper we consider a new step of this program. We discuss the scalar quantum electrodynamics with the standard $U(1)$ as the gauge group and use a quantization procedure where a deformed Heisenberg algebra describes the scalar particles and the photons are quantized in a standard way. We introduce a parameter η in those terms of the interaction Hamiltonian which have derivatives. We use this formalism to evaluate a photon-induced process of composite particles up to second order in the coupling constant. We also analyze the gauge and Lorentz symmetries of the photon-induced process. The requirement of these symmetries in the photon-induced process implies relations among the parameters of the algebra and η, leaving only one free parameter. We compare the cross-section for the scattering $2\gamma \rightarrow q$-boson $+q$-boson with the experimental data for $2\gamma \rightarrow \pi^+ + \pi^-$, where $\pi^\pm$ are the charged pions, obtaining good agreement in the region 0.55–0.7 GeV.

In section 2 we summarize the generalized Heisenberg algebra [25], slightly modified. In section 3 we describe a scalar complex quantum field theory where the scalar fields are described using the generalized Heisenberg algebra. In section 4 we introduce the interaction Hamiltonian where the complex deformed scalar fields interact with the standard electromagnetic field. Furthermore, in this section we compute perturbatively, up to second order, the interaction of one deformed boson with one standard photon giving one deformed boson and one photon and analyze the Lorentz and gauge invariance of the process. In section 5 we compute the cross-section for the scattering $2\gamma \rightarrow q$-boson $+q$-boson and compare with the experimental data for $2\gamma \rightarrow \pi^+ + \pi^-$ obtaining good agreement in the region 0.55–0.7 GeV. In section 6 we present the main differences that we have found in the perturtative computation we have performed with respect to the standard scalar QED. Moreover, we give in this section the value of the coefficient we have introduced in the interaction Hamiltonian in order to satisfy Lorentz and gauge symmetries at the quantum level. Final comments are also presented in this section. Finally, there are two appendices where we show the details of some computations.

2. Generalized Heisenberg algebra

As introduced in [25], the GHA has three generators, J_0^A, A and $A^\dagger$, and is described by the following relations:

$$J_0^A A^\dagger = A^\dagger f\left(J_0^A\right) \tag{1}$$

$$A J_0^A = f\left(J_0^A\right) A \tag{2}$$

$$[A, A^\dagger] = f\left(J_0^A\right) - J_0^A \tag{3}$$

where $f(x)$ is a general function called the characteristic function of the algebra and † means Hermitian conjugate. Each particular form of the function $f(x)$ is associated with a particular physical system; for instance, if $f(x) = qx + 1$ we have a deformed Heisenberg algebra [25], with q being the deformation parameter. Interesting physical systems, such as the square-well potential [31], the harmonic oscillator in the circle [32] and the CO molecule [13], were also analyzed through the GHA. In general, the GHA describes a class of quantum systems characterized by having successive energy eigenvalues obeying $\epsilon_{n+1} = f(\epsilon_n)$, with the function $f(x)$ being the characteristic function of the algebra.

In order to describe a complex field we must introduce a new set of operators, J_0^B, B and $B^\dagger$ satisfying similar relations,

$$J_0^B B^\dagger = B^\dagger f\left(J_0^B\right) \tag{4}$$

$$B J_0^B = f\left(J_0^B\right) B \tag{5}$$

$$[B, B^\dagger] = f\left(J_0^B\right) - J_0^B; \tag{6}$$

by hypothesis, $J_0^\dagger = J_0$ and $f(J_0)$ is an arbitrary function of J_0. Clearly, we choose

$$[A, B^\dagger] = [A^\dagger, B^\dagger] = [A^\dagger, B] = [A, B] = 0. \tag{7}$$

We assume that there is a vacuum state represented by $|0, 0\rangle$ (for simplicity, we will represent it by $|0\rangle$) and that for this vacuum state we have

$$J_0^A|0, 0\rangle = J_0^B|0, 0\rangle = \alpha_0|0, 0\rangle \equiv \alpha_0|0\rangle. \tag{8}$$

It can be shown [25] that for an arbitrary function f,

$$J_0^A|m_A, m_B\rangle = f^{(m_A)}(\alpha_0)|m_A, m_B\rangle, \tag{9}$$

$$A^\dagger|m_A, m_B\rangle = N_{m_A}|m_A + 1, m_B\rangle, \tag{10}$$

$$A|m_A, m_B\rangle = N_{m_A-1}|m_A - 1, m_B\rangle, \tag{11}$$

$$J_0^B|m_A, m_B\rangle = f^{(m_B)}(\alpha_0)|m_A, m_B\rangle, \tag{12}$$

$$B^\dagger|m_A, m_B\rangle = N_{m_B}|m_A, m_B + 1\rangle, \tag{13}$$

$$B|m_A, m_B\rangle = N_{m_B-1}|m_A, m_B - 1\rangle, \tag{14}$$

where $m_{A,B} = 1, 2, \ldots$ and

$$N_{m-1}^2 = f^{(m)}(\alpha_0) - \alpha_0, \tag{15}$$

α_0 being the lowest J_0 eigenvalue and $f^{(m)}(\alpha_0)$ is the mth iteration of α_0 through the function $f(x)$. If, for instance, $f(x) = tx^2 + qx + s$, for $m = 1$ equation (15) gives

$$N_0{}^2 = t\alpha_0{}^2 + (q - 1)\alpha_0 + s. \tag{16}$$

As noted before, GHA describes a class of quantum systems characterized by energy eigenvalues given by $\epsilon_n = f(\epsilon_{n-1})$, where ϵ_n and ϵ_{n-1} are successive energy levels. Unlike conventional Heisenberg algebra, in the general case we are considering the energy of n particles is different from n times the energy of one particle. This happens, for instance, when the ladder operators of the GHA create and/or annihilate composite particles where these composite particles are not too far from each other in order to have a small interaction among them and not too close to each other in order that each composite particle remains composite [33].

The interpretation of deformed Heisenberg algebras as describing phenomenologically composite particles is not a new idea [17–24]. The basic concept is that the deformation parameter of the algebra could incorporate the essential features of the microscopic dynamics of the physical system. As a simple example, let us consider $a_1^\dagger$ and $a_2^\dagger$ to be the creation operators of two fermions labeled by the numbers 1 and 2 and the creation operator of a composite (by these two fermions) boson-like particle as $b^\dagger = a_2^\dagger a_1^\dagger$; the annihilation operator of this boson-like particle is its adjoint. Defining the number operator of the boson-like particles as $N = \left(a_1^\dagger a_1 + a_2^\dagger a_2\right)/2$, the commutation relations of N, b and $b^\dagger$ obey a modified Heisenberg algebra. As a physical example of this, the algebra of fermion pairs with zero angular momentum can be approximated by the q-oscillator algebra [18, 19]. Also, their pairing Hamiltonian has a non-addititvity property, in the sense that the energy difference of any two successive levels are not equal. This is a property of deformed Heisenberg algebras unlike the harmonic oscillator algebra. In their shell model of nuclear collective motion [18, 19], the fermion pairs of zero angular momentum ($J = 0$) in a single j-shell are created by the pair creation operator

$$B^\dagger = \frac{1}{\sqrt{\Omega}} \sum_{m>0} (-1)^{j+m} f_{j,m}^\dagger f_{j,-m}^\dagger, \tag{17}$$

with $-j \leqslant m \leqslant j$, where $f_{j,m}^\dagger$ are fermion creation operators and $2\Omega = 2j + 1$ is the degeneracy of the shell (here we are using the same notation of [18, 19]). The pair creation and annihilation operators satisfy the algebra

$$[B, B^\dagger] = 1 - \frac{N_F}{\Omega}, \tag{18}$$

where $N_F = \sum_{m>0} \left(f_{j,m}^\dagger f_{j,m} + f_{j,-m}^\dagger f_{j,-m}\right)$ is the fermion number operator and the pairing Hamiltonian is $H = -G\Omega B^\dagger B$. In [18, 19] it is shown that q-oscillator algebra can approximate the algebra given by equation (18).

3. Complex spinless GQFT

As was noted in [34], it is possible to construct a generalized QFT with a nonlinear function $f(J_0) = tJ_0^2 + qJ_0 + s$. Obviously for $t = 0$ we have the linear case. Thus, from now on we discuss the algebra given by (1)–(3) for this linear case (and $s = 1$), which is the simplest case. In this case the algebraic relations (1)–(3) can be written as [25]

$$\left[J_0^A, A^\dagger\right]_q = A^\dagger, \tag{19}$$

$$\left[J_0^A, A\right]_{q^{-1}} = -\frac{1}{q} A, \tag{20}$$

$$[A, A^\dagger] = (q-1)J^A{}_0 + 1, \tag{21}$$

where $[a, b]_q = ab - qba$ is the q-deformed commutation relation between two operators a and b. The same relations are valid for J_0^B, B and $B^\dagger$. Equations (19–(21) describe a one-parameter deformed Heisenberg Algebra already discussed in [25]. Of course, for $q = 1$, we recover the standard Heisenberg algebra.

By defining the standard number operators N_A and N_B such that

$$N^A|m_A, m_B\rangle = m_A|m_A, m_B\rangle \tag{22}$$

and

$$N^B|m_A, m_B\rangle = m_B|m_A, m_B\rangle. \tag{23}$$

Thus, using (9)–(14) one can write J_0 as

$$J_0^{A/B} = [N^{A/B}]_q N_0^2 + \alpha_0, \tag{24}$$

where

$$[N^{A/B}]_q = \frac{q^{N^{A/B}} - 1}{q - 1} \tag{25}$$

is the standard Gauss number. Making use of the shift operator $T_{A/B}$, with $T_{A/B} = \bar{T}^{\dagger}_{A/B}$ (see [35] and [31] for more details) obeying the following relations,

$$\bar{T}_A|m_A, m_B\rangle = |m_A + 1, m_B\rangle, \tag{26}$$

$$T_A|m_A, m_B\rangle = |m_A - 1, m_B\rangle \tag{27}$$

and

$$\bar{T}_B|m_A, m_B\rangle = |m_A, m_B + 1\rangle, \tag{28}$$

$$T_B|m_A, m_B\rangle = |m_A, m_B - 1\rangle, \tag{29}$$

we rewrite the operators, A, $A^{\dagger}$ and B, $B^{\dagger}$ with the help of equations (9)–(14) as

$$A^{\dagger} = S(N^A)\bar{T}_A, \tag{30}$$

$$A = T_A S(N^A), \tag{31}$$

$$B^{\dagger} = S(N^B)\bar{T}_B, \tag{32}$$

$$B = T_B S(N^B), \tag{33}$$

where

$$S(N^{A/B})^2 = [N^{A/B}]_q N_0^2. \tag{34}$$

The foregoing algebra can be used to construct a generalized quantum field theory (GQFT) which, obviously, inherit all of its peculiarities. Using the procedure described in [26–28, 34] for the complex spinless field, we define

$$\phi(\vec{r}, t) = \sum_{\vec{k}} \frac{1}{\sqrt{2\Omega w(\vec{k})}} \left(A_{\vec{k}}\, \mathrm{e}^{-\imath\vec{k}.\vec{r}} + B^{\dagger}_{\vec{k}}\, \mathrm{e}^{\imath\vec{k}.\vec{r}}\right), \tag{35}$$

$$\phi^{\dagger}(\vec{r}, t) = \sum_{\vec{k}} \frac{1}{\sqrt{2\Omega w(\vec{k})}} \left(A^{\dagger}_{\vec{k}}\, \mathrm{e}^{\imath\vec{k}.\vec{r}} + B_{\vec{k}}\, \mathrm{e}^{-\imath\vec{k}.\vec{r}}\right), \tag{36}$$

$$\Pi^{\dagger}(\vec{r}, t) = \sum_{\vec{k}} \frac{\imath w(\vec{k})}{\sqrt{2\Omega w(\vec{k})}} \left(-A_{\vec{k}}\, \mathrm{e}^{-\imath\vec{k}.r} + B^{\dagger}_{\vec{k}}\, \mathrm{e}^{\imath\vec{k}.\vec{r}}\right), \tag{37}$$

$$\Pi(\vec{r}, t) = \sum_{\vec{k}} \frac{\imath w(\vec{k})}{\sqrt{2\Omega w(\vec{k})}} \left(-A^{\dagger}_{\vec{k}}\, \mathrm{e}^{\imath\vec{k}.\vec{r}} + B_{\vec{k}}\, \mathrm{e}^{-\imath\vec{k}.r}\right), \tag{38}$$

where the coefficients in (35)–(38) satisfy relations (19)–(21), $w(\vec{k}) = \sqrt{\vec{k}^2 + m^2}$, m is a real parameter and Ω is the volume of a rectangular box.

Inserting the Fourier expansion of the field operator into

$$\mathcal{H}_0^{\mathrm{KG}} = \frac{1}{2}\int \mathrm{d}^3\vec{r}\,(\Pi^{\dagger}(\vec{r}, t)\Pi(\vec{r}, t) + \vec{\nabla}\phi^{\dagger}(\vec{r}, t)\vec{\nabla}\phi(\vec{r}, t) + m^2\phi(\vec{r}, t)^{\dagger}\phi(\vec{r}, t)), \tag{39}$$

we find

$$\mathcal{H}_0^{\mathrm{KG}} = \frac{1}{2}\sum_{\vec{k}} w(\vec{k}) N_0^2 \left\{\left[N^A_{\vec{k}} + 1\right]_q + \left[N^A_{\vec{k}}\right]_q + \left[N^B_{\vec{k}} + 1\right]_q + \left[N^B_{\vec{k}}\right]_q\right\} \tag{40}$$

where $[N^{A/B}]_q$ is the standard Gauss number operator. Note that in the limit $q \to 1$, the Hamiltonian is proportional to the number operator.

The time evolution of the fields can be studied by solving Heisenberg's equation for $A^\dagger_{\vec{k}}, A_{\vec{k}}, B^\dagger_{\vec{k}}, B_{\vec{k}}$. Thus, using equation (40) we obtain the following results:

$$A^\dagger_{\vec{k}} = A^\dagger_{\vec{k}}(0)\,\mathrm{e}^{-\imath w(\vec{k})h(N^A_{\vec{k}})t} \tag{41}$$

$$B^\dagger_{\vec{k}} = B^\dagger_{\vec{k}}(0)\,\mathrm{e}^{-\imath w(\vec{k})h(N^B_{\vec{k}})t} \tag{42}$$

where

$$h\big(N^{A/B}_{\vec{k}}\big) = \tfrac{1}{2}\Delta E N_0^2(1+q), \tag{43}$$

and $\Delta E = \big[N^{A/B}_{\vec{k}} + 1\big]_q - \big[N^{A/B}_{\vec{k}}\big]_q$. For later use let us define the vacuum expectation value of (43) represented here by

$$h(0) = \langle 0|h\big(N^{A/B}_{\vec{k}}\big)|0\rangle = \zeta N_0^2 \tag{44}$$

where

$$\zeta = \tfrac{1}{2}(q+1). \tag{45}$$

We impose the following constraint among the parameters of GHA by choosing

$$h(0) = 1; \tag{46}$$

this constraint is necessary to preserve the Lorentz invariance of the theory. It is easy to see that the solution of this constraint is $\alpha_0 = -1/(q+1)$ which, according to [25], is only possible for $-1 < q < 1$.

Using equations (41) and (42) in the Fourier expansions shown in (35)–(38), the field $\phi(x)$ can thus be written as $(x \equiv (\vec{r}, t))$

$$\phi(x) = \alpha(x) + \beta^\dagger(x), \tag{47}$$

where

$$\alpha(x) = \sum_{\vec{k}} \frac{1}{\sqrt{2\Omega w(\vec{k})}}\,\mathrm{e}^{-\imath\vec{k}.\vec{r}+\imath w(\vec{k})h(N^A_{\vec{k}})t} A_{\vec{k}}(0) \tag{48}$$

and

$$\beta^\dagger(x) = \sum_{\vec{k}} \frac{1}{\sqrt{2\Omega w(\vec{k})}} B^\dagger_{\vec{k}}(0)\,\mathrm{e}^{\imath\vec{k}.\vec{r}-\imath w(\vec{k})h(N^B_{\vec{k}})t}. \tag{49}$$

The generalized Pauli–Jordan delta is defined as the commutator between the field operator $\phi(x, x_0)$ and $\phi^\dagger(y, y_0)$ for arbitrary, possibly unequal times x_0, y_0:

$$\Delta^N(x-y) := [\phi(x), \phi^\dagger(y)]. \tag{50}$$

The vacuum expectation value of (50) preserves all properties of the conventional Pauli–Jordan function $\Delta(x)$, for instance, the fundamental property of quantum fields

$$\langle 0|\Delta^N(x-y)|0\rangle = 0, \tag{51}$$

outside of the light cone, i.e, for space-like distances $(x-y)^2 < 0$. One can find an explicit expression for the operator $\Delta^N(x-y)$ inserting expansion (35) and the Hermitian adjoint in (50). The expression gets simplified through the use of the commutation relations (3) and (6):

$$\imath\Delta^N(x-y) = \imath\big(f\big(J_0^A\big) - J_0^A\big)\Delta^N_{(+)} + \imath\big(f\big(J_0^B\big) - J_0^B\big)\Delta^N_{(-)}, \tag{52}$$

where

$$\Delta^{N}_{(\pm)}(x-y)=\int\frac{\mathrm{d}^3p}{(2\pi)^3\omega(\vec{p})}\,\mathrm{e}^{\pm\imath\vec{p}\cdot(\vec{x}-\vec{y})\mp\imath w(\vec{k})h(N)t} \tag{53}$$

and the vacuum expectation value is given by

$$\langle 0|\Delta^N(x-y)|0\rangle = N_0^2\langle 0|\Delta(x-y)|0\rangle, \tag{54}$$

$\Delta(x-y)$ being the conventional Pauli–Jordan function.

The next task is to evaluate the Dyson–Wick field contraction between ($x_i \equiv (r_i, t_i)$) $\phi(x_1)$ and $\phi^\dagger(x_2)$, which gives

$$\imath\Delta_F^{(N^A_{\vec{k}},N^B_{\vec{k}})}(x)=\frac{\imath N_0^2}{(2\pi)^4}\int \mathrm{d}^4k\,\frac{F(N_{\vec{k}})\,\mathrm{e}^{-\imath\vec{k}.\vec{r}+\imath k_0 H(N_{\vec{k}})t}}{k^2-m^2}-(N\to N-1), \tag{55}$$

where

$$F(N_{\vec{k}})=\left[N^A_{\vec{k}}+1\right]_q\theta(k_{0x})+\left[N^B_{\vec{k}}+1\right]_q\theta(-k_{0x}) \tag{56}$$

and

$$H(N_{\vec{k}})=\left[h\left(N^A_{\vec{k}}\right)\theta(k_{0x})+h\left(N^B_{\vec{k}}\right)\theta(-k_{0x})\right], \tag{57}$$

where $\theta(x)$ is the Heaviside function. It is clear that the Dyson–Wick contraction, for instance the $\Delta_F^N(x)$ given in equation (55), in GQFT [26, 27] is not a c-number and hence it does not commute with ϕ,

$$\left[\Delta_F^N(x),\phi(x)\right]\neq 0, \tag{58}$$

as it commutes in conventional quantum field theory. As shown in [26, 27], this fact leads to a change in Wick theorem. The propagator is defined as the vacuum expectation value of equation (55),

$$\Delta_F^0(x)=\frac{N_0^2}{(2\pi)^4}\int \mathrm{d}^4k\,\frac{\mathrm{e}^{-\imath\vec{k}.\vec{r}+\imath k_0 h(0)t}}{k^2-m^2}. \tag{59}$$

Using condition (46) we obtain

$$\Delta_F^0(x)=N_0^2\Delta_F(x), \tag{60}$$

where $\Delta_F(x)$ is the usual scalar Feynman propagator.

The charge operator of this theory must obey the usual commutation rules

$$[Q,\phi(x)]=-\phi(x),\qquad [Q,\phi^\dagger(x)]=-\phi^\dagger(x), \tag{61}$$

inasmuch as the field operator $\phi^\dagger$ may increases the charge of a state by one unit and similarly the operator ϕ reduces the charge by one unit. Therefore, the charge operator is

$$Q=\int \mathrm{d}^3k\left(N^A_k-N^B_k\right). \tag{62}$$

4. Scattering of photons by composite particles

4.1. Perturbative computation and Lorentz covariance

Using the formalism we have just presented in the last section we are going now to analyze the scattering process of a charged scalar composite particle by a photon. The Hamiltonian of a scalar charged particle interacting with a photon is

$$\mathcal{H}=\mathcal{H}_0^{\mathrm{KG}}+\mathcal{H}_0^{\mathrm{em}}+\mathcal{H}_{\mathrm{int}}, \tag{63}$$

where $\mathcal{H}_0^{\mathrm{KG}}$ is given previously in equation (39), $\mathcal{H}_0^{\mathrm{em}}$ is the Hamiltonian of the Maxwell field and

$$\mathcal{H}_{\mathrm{int}} = \mathrm{i}e\phi^\star(x)\overleftrightarrow{\partial}_k\phi(x)C^k(x) + \mathrm{i}e(\pi^\star(x)\phi^\star(x) - \pi(x)\phi(x))C^0(x) \\ - e^2\phi^\star(x)\phi(x)C_\mu(x)C^\mu(x) + e^2\phi^\star(x)\phi(x)C^0(x)^2 \tag{64}$$

is the interaction Hamiltonian where the electromagnetic field is the standard one and is denoted by $C_\mu(x)$ and

$$\phi^\star(x)\overleftrightarrow{\partial}_k\phi(x) \equiv -\partial_k\phi^\star(x)\phi(x) + \phi^\star(x)\partial_k\phi(x). \tag{65}$$

In the interaction picture it is easy to see from equations (35)–(38) and (41)–(42) that

$$\pi^\dagger(\vec{r}, t) = \partial_0\phi(\vec{r}, t) + \rho(\vec{r}, t), \tag{66}$$

where

$$\rho(\vec{r}, t) = \sum_{\vec{k}} \frac{i w(\vec{k})}{\sqrt{2\Omega w(\vec{k})}} \left(M_1^k A_{\vec{k}}\, \mathrm{e}^{-\imath\vec{k}.\vec{r}} + B_{\vec{k}}^\dagger M_2^k\, \mathrm{e}^{\imath\vec{k}.\vec{r}}\right) \tag{67}$$

and

$$M_1^k = \left(1 - h\left(N_k^A\right)\right) \mathrm{e}^{\imath w(\vec{k})h(N_k^A)t}, \tag{68}$$

$$M_2^k = \left(1 - h\left(N_k^B\right)\right) \mathrm{e}^{-\imath w(\vec{k})h(N_k^B)t}. \tag{69}$$

Note that for $q \to 1$ one has $h(N) \to 1$, thus $\rho(\vec{r}, t) = 0$ and one obtains the standard result. Now, according to equations (66)–(69), the interaction Hamiltonian within the interaction picture can be written as

$$\mathcal{H}_{\mathrm{int}} = i : e\phi^\dagger(x)\overleftrightarrow{\partial}_\mu\phi(x)C^\mu : -e^2 : \phi^\dagger(x)\phi(x)C_\mu C^\mu : \\ + e^2 : \phi^\dagger(x)\phi(x)(C^0)^2 : + \mathrm{i}e : (\rho(x)\phi^\dagger(x) - \rho^\dagger(x)\phi(x))C^0 :, \tag{70}$$

where the symbol :: means the normal ordering prescription.

By hypothesis the composite particles are created by the GHA operators whereas the photon is supposed to be the standard structureless neutral particle. We shall analyze the following scattering process,

$$P^+ + \gamma \to P'^+ + \gamma', \tag{71}$$

with an initial state

$$|i\rangle \equiv |k, p\rangle = \frac{a_{k\lambda}^\dagger A_p^\dagger}{N_0}|0\rangle \tag{72}$$

and a final state

$$|f\rangle \equiv |k', p'\rangle = \frac{a_{k'\sigma}^\dagger A_{p'}^\dagger}{N_0}|0\rangle, \tag{73}$$

where $A^\dagger, A$ create and annihilate composite particles, respectively, and $a_{\vec{k}_1\lambda}^\dagger, a_{\vec{k}_1\lambda}$ are the coefficients of the Fourier expansion of the photon field given as

$$C^\mu(x) = \sum_{\vec{k}} \frac{1}{\sqrt{2\Omega\omega_k}} \sum_{\lambda=0}^{3} \left(a_{\vec{k}\lambda}\epsilon^\mu(k, \lambda)\, \mathrm{e}^{\mathrm{i}k.x} + a_{\vec{k}\lambda}^\dagger\epsilon^\mu(k, \lambda)\, \mathrm{e}^{-\mathrm{i}k.x}\right), \tag{74}$$

with $\epsilon^\mu(k, \lambda)$ being the polarization vectors and N_0 given by equation (16). For simplicity we have changed our notation and now in the ket vector $|k, p\rangle$, k represents the momentum of the photon and p stands for the momentum of the composite particle (instead of $|m_A, m_B\rangle$) and $|0\rangle$ represents the vacuum.

As was explained before, the quantization through deformed algebras we are implementing can be used to describe phenomenologically interactions of composite particles. Moreover, these composite particles being not point like ones have an extent, thus it is possible that the terms in the interacting phenomenological Hamiltonian which are proportional to derivatives differ from the standard point-like Hamiltonian by a constant η satisfying the condition

$$\lim_{q\to 1} \eta = 1. \tag{75}$$

Therefore we consider the interaction of the photon with the charged composite particle described by the following phenomenological Hamiltonian,

$$\begin{aligned}\mathcal{H}_{\text{int}} = \mathrm{i}e\eta :& [\phi^\dagger(x)\partial_\mu\phi(x) - \partial_\mu\phi(x)^\dagger\phi(x)]C^\mu : \\ & - e^2 : \phi^\dagger(x)\phi(x)C_\mu C^\mu : + e^2 : \phi^\dagger(x)\phi(x)(C^0)^2 : \\ & + \mathrm{i}e : (\rho(x)\phi^\dagger(x) - \rho^\dagger(x)\phi(x))C^0 : .\end{aligned} \tag{76}$$

Inspecting (76) we note that the last two terms are non-covariant and appear to destroy the Lorentz covariance of the theory. When working out the perturbation expansion, however, one observes that the boson propagator also contains a non-covariant part that cancels the contribution of the non-covariant term proportional to e^2. We shall see that this mechanism is preserved here. Moreover, because $h(0) = 1$, as is seen in equation (46), the last term in equation (76) will not contribute.

Let us now consider the matrix element $S^1_{fi} = \langle f|\hat{S}^1|i\rangle$, where $\hat{S}^1$ is the first order perturbative expansion of the S-matrix $\hat{S}^1 = -\mathrm{i}\int \mathrm{d}^4x\mathcal{H}_{\text{int}}$, with $\mathcal{H}_{\text{int}}$ given in equation (76) and the states $|i, f\rangle$ given in equations (72) and (73). The terms that give non-zero contributions to S^1_{fi} are

$$S^1_{fi} = \mathrm{i}e^2 \int \mathrm{d}^4x\langle f| : \phi^\dagger(x)\phi(x)C_\mu(x)C^\mu(x) : |i\rangle + ET, \tag{77}$$

where

$$ET = -\mathrm{i}e^2 \int \mathrm{d}^4x\langle f| : \phi(x)^\dagger\phi(x)(C^0)^2 : |i\rangle. \tag{78}$$

In order to compute the first term on the right-hand side of equation (77), first we insert the expansions of the field operators given in equations (35), (36) and (74) and the states given in equations (72) and (73) in the first term of equation (77). Taking into account

$$\left[a_{\vec{k}'\lambda}, a^\dagger_{\vec{k}'_3\sigma}\right] = \delta_{\vec{k}',\vec{k}_3}\delta_{\lambda,\sigma}, \tag{79}$$

also

$$\langle 0|A_{\vec{p}'}A^\dagger_{\vec{k}_1}A_{\vec{k}_2}A^\dagger_{\vec{p}}|0\rangle = N_0^4\delta_{\vec{p}',\vec{k}_1}\delta_{\vec{p},\vec{k}_2} \tag{80}$$

and that $[N, A^\dagger] = A^\dagger$, we obtain

$$S^1_{fi} = \frac{\mathrm{i}e^2N_0^2\epsilon(k',\sigma).\epsilon(k,\lambda)}{2\Omega^2\sqrt{\omega_p\omega_{p'}\omega_k\omega_{k'}}}\int \mathrm{d}^4x\, \mathrm{e}^{\mathrm{i}(\omega(p')-\omega(p))h(0)t+(\vec{p}-\vec{p}')\cdot\vec{r}+i(k-k')\cdot x} + ET, \tag{81}$$

choosing, as in equation (46), $h(0) = 1$ and carrying out the integration in the first term in equation (81) we obtain

$$\mathcal{S}^1_{fi} = \frac{\mathrm{i}e^2N_0^2\epsilon_1\cdot\epsilon_2}{4\pi^2\sqrt{\omega_p\omega_{p'}\omega_k\omega_{k'}}}\delta^{(4)}(p+k-p'-k') + ET, \tag{82}$$

where

$$\epsilon_1 \cdot \epsilon_2 = \epsilon_\mu(k', \sigma)\epsilon^\mu(k, \lambda) \tag{83}$$

and N_0 is defined in equation (16).

There is a further contribution to the element of matrix we are computing of the same order e^2, which has to be added coherently. This contribution comes from the second-order term in the perturbation series expansion of the S-matrix which is given by

$$\mathcal{S}^2_{fi} = \frac{(-i)^2}{2!}\int \mathrm{d}^4x\,\mathrm{d}^4y\langle f|T(\mathcal{H}_{\text{int}}(x)\mathcal{H}_{\text{int}}(y))|i\rangle, \tag{84}$$

where $\langle f|$ and $|i\rangle$ are given in equations (72) and (73), respectively. Inserting the interaction Hamiltonian, given in equation (76), in equation (84) and using the generalized Wick's theorem given in the appendix of [27], we obtain

$$\begin{aligned}\mathcal{S}^2_{fi} = \frac{e^2\eta^2}{2!}\int \mathrm{d}^4x\,\mathrm{d}^4y\langle f|(&\phi^\dagger(x)\partial_\mu\phi(x)\phi^\dagger(y)\partial_\nu\phi(y)\\ &+\phi^\dagger(x)\partial_\mu\phi(x)\phi^\dagger(y)\partial_\nu\phi(y) - \phi^\dagger(x)\partial_\mu\phi(x)\partial_\nu\phi(y)^\dagger\phi(y)\\ &+\phi^\dagger(x)\partial_\mu\phi(x)\partial_\nu\phi^\dagger(y)\phi(y) + \partial_\mu\phi^\dagger(x)\phi(x)\phi^\dagger(y)\partial_\nu\phi(y)\\ &+\partial_\mu\phi^\dagger(x)\phi(x)\phi^\dagger(y)\partial_\nu\phi(y) + \partial_\mu\phi^\dagger(x)\phi(x)\partial_\nu\phi^\dagger(y)\phi(y)\\ &+\partial_\mu\phi^\dagger(x)\phi(x)\partial_\nu\phi^\dagger(y)\phi(y)) : C^\mu(x)C^\nu(y) : |i\rangle.\end{aligned} \tag{85}$$

Note that we have discarded the contribution of the last term of the interaction Hamiltonian given in equation (76) because, as shown in appendix A, this term gives no contribution for the present computation. Moreover, since the contractions in the above equation now are not c-numbers each term must be considered separately.

To explicitly evaluate equation (85) we might first derive the action of gradient operators on the Dyson–Wick contractions of two boson field operator

$$\imath\Delta^N_F(x-y) = \phi(x)\phi^\dagger(y) = [\alpha(x), \alpha^\dagger(y)]\theta(x_0 - y_0) + [\beta(y), \beta^\dagger(x)]\theta(y_0 - x_0). \tag{86}$$

Thus, the action of a single gradient operator on the Dyson–Wick contraction gives

$$\imath\partial^y_\nu\Delta^N_F(x-y) = \phi(x)\partial^y_\nu\phi^\dagger(y) - g_{\nu 0}\Delta^N(x-y)\delta(x_0 - y_0), \tag{87}$$

where $\Delta^N(x-y)$ is the generalized Pauli–Jordan delta (50). The last term in the above equation gives no contribution when inserted into an element of matrix of the S-matrix. For instance, the computation of

$$\delta(x_0 - y_0)\langle f|\Delta^N(x-y)|i\rangle, \tag{88}$$

where the states are given in equations (72) and (73), gives

$$\delta(x_0 - y_0)\Delta^{(\delta_{k,p})}(x-y) = N_0^2\delta(x_0 - y_0)\Delta(x-y) = 0, \tag{89}$$

where $\Delta(x-y)$ is the standard Pauli–Jordan function, and $\vec{k}$ and $\vec{p}$ are the momentum in the integral representation of the generalized Pauli–Jordan function and the initial momentum of the composite particle. The first equality in the above equation is obtained by using the property

$$\int_{-\infty}^{\infty} \mathrm{d}x\, f(x + \delta_{x,x_0}) = \int_{-\infty}^{\infty} \mathrm{d}x\, f(x) \tag{90}$$

inside the definition of the generalized Pauli–Jordan function equation (53). However, if a second gradient operator acts,

$$\imath\partial^x_\mu\partial^y_\nu\Delta^N_F(x-y) = \partial^x_\mu\overbracket{\phi(x)\partial^y_\nu\phi^\dagger}(y) + g_{\mu 0}\partial^y_\nu\Delta^N(x-y)\delta(x_0-y_0), \tag{91}$$

unlike equation (87) the last term of (91) does not vanish when inserted inside the initial and final states. From (86), (87), (91) we conclude that

$$\begin{aligned}&\overbracket{\phi(x)\phi^\dagger}(y) = \Delta^N_F(x-y)\\ &\overbracket{\phi(x)\partial^y_\nu\phi^\dagger}(y) = \mathrm{i}\partial^y_\nu\Delta^N_F(x-y) + g_{\nu 0}\Delta^N(x-y)\delta(x_0-y_0)\end{aligned} \tag{92}$$

and

$$\partial_\mu\overbracket{\phi(x)\partial_\nu\phi^\dagger}(y) = \mathrm{i}\partial^x_\mu\partial^y_\nu\Delta^N_F(x-y) - \imath g_{\mu 0}\partial^y_\nu\Delta^N(x-y)\delta(x_0-y_0). \tag{93}$$

Now, let us go back to the computation of equation (85). As is shown in the appendix of [27], because the contractions of the deformed fields are not *c*-numbers Wick's theorem has an additional subtlety and the explicit expressions of time-ordered products are less simple in this case than the standard case. However, as shown in appendix B of this paper, taking the matrix elements of time-ordered products of fields and their derivatives because of the property given in equation (90) we recover the standard expression of Wick's theorem with the following modifications:

$$\begin{aligned}&\Delta^N_F(x-y)\longrightarrow\Delta^0_F(x-y) = N_0^2\Delta_F(x-y),\\ &\Delta^N(x-y)\longrightarrow\Delta^0(x-y) = N_0^2\Delta(x-y),\end{aligned} \tag{94}$$

where $\Delta_F(x-y)$ and $\Delta(x-y)$ are the conventional scalar Feynman propagator and Pauli–Jordan function, respectively. Thus $\mathcal{S}^2_{fi}$ becomes

$$\begin{aligned}\mathcal{S}^2_{fi} = \imath e^2\eta^2\int \mathrm{d}^4x\,\mathrm{d}^4y\langle f|:&\big[\partial^x_\mu\Delta^0_F(x-y)\phi^\dagger(x)\partial_\nu\phi(y)\\ &-\Delta^0_F(x-y)\partial^x_\mu\phi^\dagger(x)\partial^y_\nu\phi(y) - \partial^x_\mu\partial^y_\nu\Delta^0_F(x-y)\phi^\dagger(x)\phi(y)\\ &+\partial^y_\nu\Delta^0_F(x-y)\partial^x_\mu\phi^\dagger(x)\phi(y)\big]::C^\mu(x)C^\nu(y):|i\rangle\\ &+\imath e^2\eta^2\int\mathrm{d}^4x\,\mathrm{d}^4y\langle f|:\delta(x_0-y_0)\partial^y_\nu\Delta^0_F(x-y)\phi^\dagger(x)\phi(y):\\ &\times\,:C_0(x)C^\nu(y):|i\rangle.\end{aligned} \tag{95}$$

Since $\imath\delta(x_0-y_0)\partial^y_\nu\Delta^0(x-y) = \imath N_0^2 g_{\nu 0}\delta^4(x-y)$ the last term in (95) gives

$$\begin{aligned}&\imath e^2\eta^2N_0^2\int\mathrm{d}^4x\,\mathrm{d}^4y\langle f|:\phi^\dagger(x)\phi(y)\,(C_0(x))^2:\delta^4(x-y)|i\rangle\\ &\quad=\imath e^2\eta^2N_0^2\int\mathrm{d}^4x\langle f|:\phi^\dagger(x)\phi(y)\,(C_0(x))^2:|i\rangle.\end{aligned} \tag{96}$$

Then we see that if we choose

Lorentz invariance condition

$$\eta^2 = \frac{1}{N_0^2}, \tag{97}$$

the last term in equation (95) cancels the term we called ET given in equation (78). Thus, with this choice, we have shown that up to order e^2 the scattering of photon by a composite particle described in (71)–(73) is Lorentz invariant.

The remaining terms in equation (95) give the additional terms to be added to equation (82) to obtain the total matrix element of the scattering described in (71)–(73) up to order e^2. Let us start considering the first term in equation (95), i.e.,

$$T_1 = \frac{\imath e^2\eta^2}{N_0^2}\int \mathrm{d}^4x\,\mathrm{d}^4y\,\partial^x_\mu\Delta^0_F(x-y)\langle 0|A_{p'}:\phi^\dagger(x)\partial^y_\nu\phi(y):A^\dagger_p|0\rangle \\ \times\langle 0|a_{k'\sigma}:C^\mu(x)C^\nu(y):a^\dagger_{k\lambda}|0\rangle. \tag{98}$$

The only part of this computation which is different from the usual evaluation in quantum field theory is the computation of the matrix element $\langle 0|A_{p'}:\phi^\dagger(x)\partial^y_\nu\phi(y):A^\dagger_p|0\rangle$. Using equations (35), (36), (46) and (80), we easily find

$$\langle 0|A_{p'}:\phi^\dagger(x)\partial^y_\nu\phi(y):A^\dagger_p|0\rangle = \mathrm{e}^{\imath\vec{k}_3.\vec{x}-\imath\omega(k_3)h(-1+\delta_{\vec{p}',\vec{k}_3})t_x} \\ \partial^y_\nu\,\mathrm{e}^{-\imath\vec{k}_4.\vec{y}+\imath\omega(k_4)h(-1+\delta_{\vec{p},\vec{k}_4})t_y}\langle 0|A_{p'}A^\dagger_{k_3}A_{k_4}A^\dagger_p|0\rangle = -\imath N_0^4 p_\nu\,\mathrm{e}^{\imath(p'.x-p.y)}\delta_{\vec{p}',\vec{k}_3}\delta_{\vec{p},\vec{k}_4}. \tag{99}$$

The rest of the evaluation of T_1 is completely similar to the standard case and the final result is

$$T_1 = \frac{-\imath e^2\eta^2(2\pi)^4N_0^4\epsilon^\mu(k',\sigma)\epsilon^\nu(k,\lambda)}{4\Omega^2\omega(p)\omega(p')\omega(k)\omega(k')} \\ \times\left[\frac{p_\nu(p+k)_\mu}{(p+k)^2-m^2}+\frac{p_\mu(p'-k)_\nu}{(p'-k)^2-m^2}\right]\delta^{(4)}(p'+k'-p-k). \tag{100}$$

Proceeding in the same way in the computation of the other terms equation (95), puting the final result together with those obtained in equation (82) in the first order perturbative expansion for the S-matrix, we finally obtain, using the Lorentz invariant choice $\eta^2 = 1/N_0^2$,

$$\mathcal{S}_{fi} = \frac{\mathrm{i}e^2N_0^2\epsilon^\mu(k',\sigma)\epsilon^\nu(k,\lambda)}{16\pi^2\sqrt{\omega(p)\omega(p')\omega(k)\omega(k')}}T_{\mu\nu}\delta^4(p+k-p'-k'), \tag{101}$$

where

$$T_{\mu\nu} = \left[\frac{M_{\mu\nu}}{(p+k)^2-m^2}+\frac{N_{\mu\nu}}{(p'-k)^2-m^2}\right]+2g_{\mu\nu} \tag{102}$$

and

$$M_{\mu\nu} = -[p_\nu+(p+k)_\nu][(p+k)_\mu+p'_\mu], \tag{103}$$

$$N_{\mu\nu} = -[p_\mu+(p'-k)_\mu][(p'-k)_\nu+p'_\nu]. \tag{104}$$

4.2. Gauge invariance

The classical Hamiltonian with the parameter η, see (39) and (76), is not gauge invariant. It is interesting to note that the amplitude of scattering we have computed, (101)–(104), is gauge invariant if the values of this parameter is chosen in order to satisfy Lorentz invariance. Of course, we impose that the amplitude is not changed when the potential is re-gauged, $\epsilon_\mu(k)\to\epsilon_\mu(k)+k_\mu\Lambda(k)$, i.e.,

$$T_{\mu\nu}k^\nu = k'^\mu T_{\mu\nu} = 0. \tag{105}$$

Rewriting the dominator in (102) as

$$(p+k)^2-m^2 = p^2+2k\cdot p+k^2-m^2 = 2k\cdot p,$$
$$(p'-k)^2-m^2 = p'^2+2k\cdot p'-k^2-m^2 = -2k\cdot p',$$

and computing $M_{\mu\nu}k_\nu$ and $N_{\mu\nu}k_\nu$, we obtain easily that the condition given in (105) is trivially satisfied.

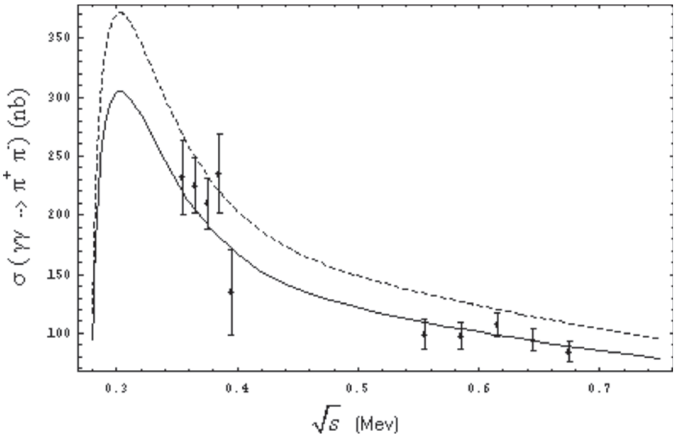

Figure 1. Experimental data and a comparison between the cross-section predicted by the conventional QFT (dashed line) and the GQFT (solid line). The fitting was obtained for $q = q_\pi = 0.81$. The value of Z was taken as $Z = 0.6$.

5. Electromagnetic processes in relativistic meson–meson collisions

We have computed perturbatively the scattering of scalar charged q-particles by standard photons. Following [18], where it was suggested that deformed algebras could be used as a phenomenological description of composite particles, we are going to compute in this section the cross-section for the process $2\gamma \rightarrow P_q^+ + P_q^-$ where $P_q^\pm$ are the scalar charged q-particles which gives a phenomenological description of charged scalar composite particles. Moreover, we compare this cross-section with the experimental data [36] for the two-photon production of $\pi^+\pi^-$ pair where $\pi^\pm$ are the charged pions.

It was shown in the last two sections that requiring gauge and Lorentz symmetries at quantum level the S-matrix for the scattering process up to second order in the coupling constant is given by equations (101)–(104). Thus, following the procedure in [37, 38], the cross-section for the process $2\gamma \rightarrow P_q^+ + P_q^-$ is

$$\sigma_Z = \frac{\pi\alpha^2 N_0^4}{m^2}\left[2Zx(1+x)\sqrt{1-x}\left(\frac{x^2}{1-(1-x)Z^2}+1\right) - x^2(2-x)\ln\left(\frac{1+\sqrt{1-x}Z}{1-\sqrt{1-x}Z}\right)\right], \tag{106}$$

where α is the fine structure constant, $x \equiv 4m_\pi^2/s$, $s = E_{\text{c.m.}}^2$ is the Mandelstam variable, m is the mass of the pion and Z is finite solid angle ($|\cos\theta| < Z$). Note that in order to compare with experimental data we have integrated over a finite solid angle.

Figure 1 shows a comparison between the predicted cross-section of the conventional QFT and the deformed QFT for $\gamma\gamma \rightarrow \pi^+\pi^-$ along with experimental data [36, 39]. Conventional QFT describes well the experimental data at low energy range since the pion should be considered as point particles. As the energy increases conventional QFT starts to have poor agreement with experimental data. In the region 0.55–0.7 GeV the pions should not be considered point-like particles anymore and deformed QFT for $q = q_\pi = 0.81$ describes the experimental data much better. It is also interesting to mention that in this energy region perturbative QCD calculations are in poor agreement with experimental data [40].

6. Analysis of the results

We have constructed deformed scalar quantum electrodynamics where the scalar bosons are created and/or annihilated by a generalized Heisenberg algebra and the photons are described in a standard way. Following the suggestion of [17–27], we interpret the ladder operators of a deformed Heisenberg algebra as creating and/or annihilating composite particles.

The interaction part of the Hamiltonian is slightly modified: we introduce one constant, η, in those terms of the interaction Hamiltonian which have derivatives (see equation (76)). We have found for the scattering process $P^+ + \gamma \rightarrow P'^+ + \gamma'$ that for $\eta^2 = 1/N_0^2$ the model preserves Lorentz and gauge symmetries at the quantum level.

Within the framework of this deformed scalar QED we have computed, up to second order in the coupling constant, the scattering process $P^+ + \gamma \rightarrow P'^+ + \gamma'$ with an initial state

$$|i\rangle \equiv |k, p\rangle = \frac{a^{\dagger}_{k\lambda} A^{\dagger}_{p}}{N_0}|0\rangle \tag{107}$$

and a final state

$$|f\rangle \equiv |k', p'\rangle = \frac{a^{\dagger}_{k'\sigma} A^{\dagger}_{p'}}{N_0}|0\rangle, \tag{108}$$

where the bosons, denoted by P^+, are described by a GHA and the photons, denoted by γ, are described as usual.

The computation of the scattering mentioned above can be summarized in the following way. Considering the parameter η appearing in the interaction term satisfying $\eta^2 = 1\big/N_0^2$, where N_0 is defined in equation (16) for $t = 0$, the above scattering was shown to be Lorentz and gauge invariant. That is, gauge invariance is recovered at quantum level. There are two possible solutions but only the positive solution $\eta^{(+)} = 1/N_0$ satisfies equation (75). Thus, the following interaction Hamiltonian can be used

$$\mathcal{H}_{\text{int}} = \frac{ie}{N_0} : [\phi^{\dagger}(x)\partial_{\mu}\phi(x) - \partial_{\mu}\phi(x)^{\dagger}\phi(x)]C^{\mu} : -e^2 : \phi^{\dagger}(x)\phi(x)C_{\mu}C^{\mu} :, \tag{109}$$

with the standard Wick's theorem together with the following modifications:

$$\begin{aligned} \Delta^N_F(x-y) &\longrightarrow \Delta^0_F(x-y) = N_0^2\Delta_F(x-y), \\ \Delta^N(x-y) &\longrightarrow \Delta^0(x-y) = N_0^2\Delta(x-y), \end{aligned} \tag{110}$$

where $\Delta_F(x-y)$ and $\Delta(x-y)$ are the conventional scalar Feynman propagator and Pauli–Jordan function, respectively. Moreover, it should be stressed that the matrix elements of the creation and/or annihilation operators of composite particles are computed using the GHA presented in equations (1)–(3). For the scattering under consideration we have also verified that even in third order the gauge and Lorentz symmetries are preserved assuming the same relations among the parameters found in previous orders. We believe that this scheme will be preserved in all orders. It would be also interesting if we could prove that these results are general, i.e., valid for any scattering considered within this model.

In addition, it is worth noting that the classical Hamiltonian we have modified through the introduction of the constant η in the derivative terms of the interaction term is not gauge invariant anymore. However, we have also shown that the scattering mentioned above, $P^+ + \gamma \rightarrow P'^+ + \gamma'$, is gauge invariant if the constant η satisfies the Lorentz invariance condition $\eta = 1/N_0$. We have also computed the cross-section for the process $2\gamma \rightarrow q$-boson $+q$-boson for $q = q_{\pi} = 0.81$, compared with the cross-section predicted by conventional QED and the experimental data for the scattering $2\gamma \rightarrow \pi^+ + \pi^-$, where $\pi^{\pm}$ are the charged pions, obtaining good agreement in the region 0.55–0.7 GeV. The parameter q is a phenomenological parameter that could incorporate the essential features of the microscopic dynamics of a composite particle. A possible quantitative measure of the depart from a point-like particle could be seen by a parameter $n_c = |q-1|$ that gives a phenomenological account for the finite extent of the composite particle. Clearly, when $n_c = 0$ we have a point-like particle and when n_c is different from zero the particle should be considered as composite.

It would be interesting also to compare the cross-section for the scattering $2\gamma \rightarrow q$-boson $+q$-boson obtained here with the two-photon production of other mesonic pairs. We think that this cross-section will describe the experimental data in the range of energy where the charged mesons start not to be considered point-like particles anymore up to certain value of energy, having a different value of q for each different charged meson.

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Appendix A. Contribution to S-matrix from $\rho(x)$

In this appendix we are going to show that the last term of the interaction Hamiltonian given in equation (76) does not contribute to the S-matrix of the scattering of photon by a composite particle computed up to second order in the coupling constant. The contribution to S^2_{fi} of the last term of the interaction Hamiltonian is

$$\mathcal{S}^2_{fi} = \frac{(-i)^4 e^2}{2!} \int \mathrm{d}^4x\, \mathrm{d}^4y \langle f|T(: (\rho(x)\phi^\dagger(x) - \rho^\dagger(x)\phi(x)) :: \\ \times (\rho(y)\phi^\dagger(y) - \rho^\dagger(y)\phi(y)) :: C^0(x)C^0(y) :)|i\rangle, \tag{A.1}$$

where $\phi(x)$ is defined in equations (47)–(49), $\rho(x) = \rho_A(x) + \rho_{B^\dagger}(x)$,

$$\rho_A(x) = \sum_{\vec{k}} \frac{i w(\vec{k})}{\sqrt{2\Omega w(\vec{k})}} M^k_1 A_{\vec{k}}\, \mathrm{e}^{-\imath \vec{k}.\vec{r}} \tag{A.2}$$

$$\rho_{B^\dagger}(x) = \sum_{\vec{k}} \frac{i w(\vec{k})}{\sqrt{2\Omega w(\vec{k})}} B^\dagger_{\vec{k}} M^k_2\, \mathrm{e}^{\imath \vec{k}.\vec{r}} \tag{A.3}$$

with M^k_1 and M^k_2 defined in equations (68)–(69) and the initial and final states given in equations (72)–(73). We are going to consider part of the above element of matrix related to the composite particle.

It is straightforward to see, for $t_x > t_y$, that the terms which contribute to the matrix element, using the generalized Wick theorem [27], are

(*a*) $$\langle 0|A_{p'} T(: \rho(x)\phi^\dagger(x) :: \rho(y)\phi^\dagger(y) :) A^\dagger_p|0\rangle \\ = \langle 0|A'_p(\alpha^\dagger(x)\rho_A(x)\alpha^\dagger(y)\rho_A(y) + \rho_A(x)\beta(x)\rho_{B^\dagger}(y)\alpha^\dagger(y))A_p|0\rangle \tag{A.4}$$

(*b*) $$\langle 0|A_{p'} T(: \rho(x)\phi^\dagger(x) :: \rho^\dagger(y)\phi(y) :) A^\dagger_p|0\rangle \\ = \langle 0|A'_p\big(\alpha^\dagger(x)\rho_A(x)\rho^\dagger_A(y)\alpha(y) + \rho_A(x)\beta(x)\rho^\dagger_A(y)\beta^\dagger(y)\big)A_p|0\rangle \tag{A.5}$$

(*c*) $$\langle 0|A_{p'} T(: \rho^\dagger(x)\phi(x) :: \rho(y)\phi^\dagger(y) :) A^\dagger_p|0\rangle \\ = \langle 0|A'_p\big(\rho^\dagger_A(x)\alpha(x)\alpha^\dagger(y)\rho_A(y) + \rho^\dagger_{B^\dagger}(x)\rho_{B^\dagger}(y)\alpha(x)\alpha^\dagger(y)\big)A_p|0\rangle \tag{A.6}$$

$$(d)\quad \langle 0|A_{p'}T(:\rho^\dagger(x)\phi(x)::\rho^\dagger(y)\phi(y):)A_p^\dagger|0\rangle$$
$$= \langle 0|A'_p\big(\rho_A^\dagger(x)\alpha(x)\rho_A^\dagger(y)\alpha(y)+\rho_{B^\dagger}^\dagger(x)\rho_A^\dagger(y)\alpha(x)\beta^\dagger(y)\big)A_p|0\rangle. \tag{A.7}$$

After a straightforward computation, we have for the first term of the rhs of equation (A.4)

$$\langle 0|A_{p'}\alpha^\dagger(x)\rho_A(x)\alpha^\dagger(y)\rho_A(y)A_p^\dagger|0\rangle$$
$$= \sum_{k_1\cdots k_4}\frac{(i)^2\omega_{k_2}\omega_{k_4}}{Z_{k_1}\cdots Z_{k_4}}\,\mathrm{e}^{\mathrm{i}\vec{k}_1\cdot\vec{x}-\mathrm{i}\omega_{k_1}h(\delta_{k_1p'})t_x}\big(1-h\big(\delta_{k_2p'}-\delta_{k_1k_2}\big)\big)$$
$$\times\,\mathrm{e}^{-\mathrm{i}\vec{k}_2\cdot\vec{x}+\mathrm{i}\omega_{k_2}h(\delta_{k_2p'}-\delta_{k_1k_2})t_x}\langle 0|A_{p'}A_{k_1}^\dagger A_{k_2}A_{k_3}^\dagger A_{k_4}A_p^\dagger|0\rangle$$
$$\times\,\mathrm{e}^{\mathrm{i}\vec{k}_3\cdot\vec{y}-\mathrm{i}\omega_{k_3}h(\delta_{k_3p}-\delta_{k_3k_4})t_y}\big(1-h\big(\delta_{k_4p}\big)\big)\,\mathrm{e}^{-\mathrm{i}\vec{k}_4\cdot\vec{y}+\mathrm{i}\omega_{k_4}h(\delta_{k_4p})t_y}, \tag{A.8}$$

where $Z_k\equiv\sqrt{2\Omega\omega(\vec{k})}$. But since $\langle 0|A_{p'}A_{k_1}^\dagger A_{k_2}A_{k_3}^\dagger A_{k_4}A_p^\dagger|0\rangle = N_0^6\delta_{p'k_1}\delta_{k_2k_3}\delta_{k_4p}$ summing over k_1 the above expression we have $h\big(\delta_{k_2p'}-\delta_{k_1k_2}\big)=h(0)=1$ and thus the above term, (A.8), is identically null.

The second term of the rhs of equation (A.4) can be separated, for simplicity, in two parts involving the operators A, B and their adjoints. The part involving the operators A and $A^\dagger$ can be written as

$$\langle 0|A_{p'}\rho_A(x)\alpha^\dagger(y)A_p^\dagger|0\rangle = \sum_{k_1,k_2}\frac{i\omega_{k_1}}{Z_{k_1}Z_{k_2}}\,\mathrm{e}^{-\mathrm{i}\vec{k}_1\cdot\vec{x}+\mathrm{i}\omega_{k_1}h(\delta_{k_1p'})t_x}\big(1-h\big(\delta_{k_1p'}\big)\big)$$
$$\times\,\mathrm{e}^{\mathrm{i}\vec{k}_2\cdot\vec{y}-\mathrm{i}\omega_{k_2}h(\delta_{k_2p})t_y}\langle 0|A_{p'}A_{k_1}A_{k_2}^\dagger A_p^\dagger|0\rangle. \tag{A.9}$$

Since $\langle 0|A_{p'}A_{k_1}A_{k_2}^\dagger A_p^\dagger|0\rangle = N_0^4\delta_{p'k_2}\delta_{k_1p}$, it is easy to see that the above matrix element is null as $h(0)=1$. The part involving the operators B and $B^\dagger$ is also null as

$$\langle 0|\beta(x)\rho_{B^\dagger}(y)|0\rangle = \sum_{k_3k_4}\frac{i\omega_{k_4}}{Z_{k_3}Z_{k_4}}\,\mathrm{e}^{-\mathrm{i}k_3\cdot x+ik_4\cdot y}(1-h(0))\langle 0|B_{k_3}B_{k_4}^\dagger|0\rangle = 0, \tag{A.10}$$

because $h(0)=1$. We have calculated all remaining terms and all of them are identically null by similar reasons ($h(0)=1$ coming from ρ_A or/and ρ_B operators).

Appendix B. Element of matrix of time-ordered product of fields

In this appendix we are going to show that element of matrix of typical time-ordered product of fields and derivatives of fields recover the standard expression of Wick's expansion with the modifications given in equation (94). We will consider three cases.

(1) Product of fields without derivatives.

We start considering a typical term of the time ordered product

$$T(:\phi(x)\phi^\dagger(x)::\phi(y)\phi^\dagger(y):)$$

where $\phi(x)$ is given in equations (47)–(49). Taking, for instance, $t_x\geqslant t_y$ we have

$$T(\alpha^\dagger(x)\alpha(x)\alpha^\dagger(y)\alpha(y)) = \alpha^\dagger(x)\alpha(x)\alpha^\dagger(y)\alpha(y)$$
$$= \alpha^\dagger(x)\alpha^\dagger(y)\alpha(x)\alpha(y)+\alpha^\dagger(x)\Delta_F^N(x-y)\alpha(y). \tag{B.1}$$

Thus, taking the matrix element the above time-ordered product we have ($|i\rangle = A_p^\dagger|0\rangle$ and $|f\rangle = A_{p'}^\dagger|0\rangle$)

$$\langle f|T(\alpha^\dagger(x)\alpha(x)\alpha^\dagger(y)\alpha(y))|i\rangle = \langle f|\alpha^\dagger(x)\Delta_F^N(x-y)\alpha(y)|i\rangle$$
$$= \Delta_F^{(-\delta_{k,k_y}+\delta_{k,p})}(x-y)\langle f|\alpha^\dagger(x)\alpha(y)|i\rangle, \tag{B.2}$$

where in the above equation k is the internal momentum of $\Delta_F^N(x-y)$ (see equation (55)) and k_y is the momentum of the Fourier expansion of the field $\phi(y)$. But using equation (80) we have trivially

$$\begin{aligned}\langle f|T(\alpha^\dagger(x)\alpha(x)\alpha^\dagger(y)\alpha(y))|i\rangle &= \Delta_F^0(x-y)\langle f|\alpha^\dagger(x)\alpha(y)|i\rangle \\ &= N_0^2\Delta_F(x-y)\langle f|\alpha^\dagger(x)\alpha(y)|i\rangle.\end{aligned}\tag{B.3}$$

The same thing will happen for the other terms of the element of matrix of the time-ordered product $T(:\phi(x)\phi^\dagger(x)::\phi(y)\phi^\dagger(y):)$ and we have the same result as one has for the standard case with

$$\Delta_F^N(x-y)\longrightarrow\Delta_F^0(x-y)=N_0^2\Delta_F(x-y).$$

(2) Product of fields with one field with derivative.

Now we consider a typical term of the time ordered product

$$T\big(:\phi(x)\phi^\dagger(x)::\phi(y)\partial_\mu^y\phi^\dagger(y):\big)$$

where $\phi(x)$ is given in equations (47)–(49). Taking, for instance, $t_x\geqslant t_y$ we have

$$\begin{aligned}T\big(\alpha^\dagger(x)\alpha(x)\partial_\mu^y\alpha^\dagger(y)\alpha(y)\big) &= \alpha^\dagger(x)\alpha(x)\partial_\mu^y\alpha^\dagger(y)\alpha(y) \\ &= \alpha^\dagger(x)\partial_\mu^y\alpha^\dagger(y)\alpha(x)\alpha(y)+\alpha^\dagger(x)\partial_\mu^y\Delta_F^N(x-y)\alpha(y).\end{aligned}\tag{B.4}$$

Thus, taking the matrix element the above time-ordered product we have ($|i\rangle = A_p^\dagger|0\rangle$ and $|f\rangle = A_{p'}^\dagger|0\rangle$)

$$\begin{aligned}\langle f|T\big(\alpha^\dagger(x)\alpha(x)\partial_\mu^y\alpha^\dagger(y)\alpha(y)\big)|i\rangle &= \langle f|\alpha^\dagger(x)\partial_\mu^y\Delta_F^N(x-y)\alpha(y)|i\rangle \\ &= N_0^2\partial_\mu^y\Delta_F(x-y)\langle f|\alpha^\dagger(x)\alpha(y)|i\rangle,\end{aligned}\tag{B.5}$$

by the same reason as the previous case. Again, the same thing happens for the other terms of the element of matrix of the time-ordered product.

(3) Two fields with one derivative.

Considering now a typical term of the time ordered product

$$T\big(:\partial_\mu^x\phi(x)\phi^\dagger(x)::\phi(y)\partial_\mu^y\phi^\dagger(y):\big)$$

and taking, for instance, $t_x\geqslant t_y$, we have for a typical term of this expansion

$$\begin{aligned}T\big(\alpha^\dagger(x)\partial_\mu^x\alpha(x)\partial_\mu^y\alpha^\dagger(y)\alpha(y)\big) &= \alpha^\dagger(x)\partial_\mu^x\alpha(x)\partial_\mu^y\alpha^\dagger(y)\alpha(y) \\ &= \alpha^\dagger(x)\partial_\mu^y\alpha^\dagger(y)\partial_\mu^x\alpha(x)\alpha(y)+\alpha^\dagger(x)\partial_\mu^x\overbracket{\phi(x)\partial_\nu^y\phi}^\dagger(y)\alpha(y),\end{aligned}\tag{B.6}$$

but we know that

$$\partial_\mu\overbracket{\phi(x)\partial_\nu\phi}^\dagger(y) = \mathrm{i}\partial_\mu^x\partial_\nu^y\Delta_F^N(x-y)-ig_{\mu0}\partial_\nu^y\Delta^N(x-y)\delta(x_0-y_0).\tag{B.7}$$

Thus, taking the matrix element of (B.6), where as before $|i\rangle = A_p^\dagger|0\rangle$ and $|f\rangle = A_{p'}^\dagger|0\rangle$, using (B.7) and equation (80), we obtain

$$\begin{aligned}&\langle f|T\big(\alpha^\dagger(x)\partial_\mu^x\alpha(x)\partial_\nu^y\alpha^\dagger(y)\alpha(y)\big)|i\rangle = N_0^2\partial_\mu^x\partial_\nu^y\Delta_F(x-y) \\ &\langle f|\alpha^\dagger(x)\alpha(y)|i\rangle - g_{\mu,0}N_0^2\partial_\nu^y\Delta(x-y)\langle f|\alpha^\dagger(x)\alpha(y)|i\rangle.\end{aligned}\tag{B.8}$$

Again, the same thing happens for the other terms of the element of matrix of the time-ordered product.

References

[1] Kulish P and Reshetikhin N 1980 *Zapiski Nauch. Semin. LOMI* **101** 112
Kulish P and Reshetikhin N 1983 *J. Sov. Math.* **23** 2435 (Engl. transl.)
[2] Drinfel'd V G 1985 *Sov. Math.—Dokl.* **32** 254
[3] Jimbo M 1985 *Lett. Math. Phys.* **10** 63
[4] Martin-Delgado M A 1991 *J. Phys. A: Math. Gen.* **24** L1285
[5] Zachos C 1992 *Contemp. Math.* **134** 351
[6] Gupta R K, Cseh J, Ludu A and Greiner W 1992 *J. Phys. G: Nucl. Part. Phys.* **18** L73
[7] Chaichian M, Felipe R G and Montonen C 1993 *J. Phys. A: Math. Gen.* **26** 4017
[8] Monteiro M R, Rodrigues L M C S and Wulck S 1996 *Phys. Rev. Lett.* **76** 1098
[9] Kundu A 1999 *Phys. Rev. Lett.* **82** 3936
[10] Gavrilik A M 2001 *Nucl. Phys.* B **102–103** (Proc. Suppl.) 298
[11] Angelova M, Dobrev V K and Frank A 2001 *J. Phys. A: Math. Gen.* **34** L503
[12] Ballesteros A, Civitarese O and Reboiro M 2003 *Phys. Rev.* C **68** 044307
[13] de Souza J, Oliveira-Neto N M and Ribeiro-Silva C I 2006 *Eur. Phys. J.* D **40** 205
[14] Arik M and Coon D D 1976 *J. Math. Phys.* **17** 524
[15] Macfarlane A J 1989 *J. Phys. A: Math. Gen.* **22** 4581
[16] Biedenharn L C 1989 *J. Phys. A: Math. Gen.* **22** L873
[17] Greenberg O W 1991 *Phys. Rev.* D **43** 4111
[18] Bonatsos D 1992 *J. Phys. A: Math. Gen.* **25** L101
[19] Bonatsos D, Daskaloyannis C and Faessler A 1994 *J. Phys. A: Math. Gen.* **27** 1299
[20] Avancini S S and Krein G 1995 *J. Phys. A: Math. Gen.* **28** 685
[21] Raychev P P, Roussev R P, Iudice N Lo and Terziev P A 1998 *J. Phys. G: Nucl. Part. Phys.* **24** 1931
[22] Anchishkin D V, Gavrilik A M and Iorgov N Z 2000 *Eur. Phys. J.* A **7** 229
[23] Bouldjedri A and Zerguine S 2002 *J. Phys. G: Nucl. Part. Phys.* **28** 1433
[24] Sviratcheva K D, Bahri C, Georgieva A I and Draayer J P 2004 *Phys. Rev. Lett.* **93** 15201
[25] Curado E M F and Rego-Monteiro M A 2001 *J. Phys. A: Math. Gen.* **34** 3253
[26] Bezerra V B, Curado E M F and Rego-Monteiro M A 2002 *Phys. Rev.* D **65** 065020
[27] Bezerra V B, Curado E M F and Rego-Monteiro M A 2002 *Phys. Rev.* D **66** 085013
[28] Bezerra V B, Curado E M F and Rego-Monteiro M A 2004 *Phys. Rev.* D **69** 085003
[29] Lee T D 1981 *Particle Physics and Introduction to Field Theory* (New York: Harwood)
[30] Reinhardt Greiner 1996 *Field Quantization* (Berlin: Springer)
[31] Curado E M F, Rego-Monteiro M A and Nazareno H N 2001 *Phys. Rev.* A **64** 012105
[32] Rego-Monteiro M A 2001 *Eur. Phys. J.* C **21** 749
[33] See for instance Klein A and Marshalek E R 1991 *Rev. Mod. Phys.* **63** 375
[34] Ribeiro-Silva C I and Oliveira-Neto N M 2004 *Preprint* hep-ph/0409267
[35] Dimakis A and Muller-Hoissen F 1992 *Phys. Let.* B **295** 242
[36] Boyer J *et al* 1990 *Phys. Rev.* D **42** 1350
Boyer J *et al* 1992 *Phys. Rev.* B **295** 242
[37] Low F E 1954 *Phys. Rev.* **96** 1428
[38] Blundell H G, Godfrey S, Hay G and Swanson S 2000 *Phys. Rev.* C **61** 025203
[39] Donoghue J F and Holstein B R 1993 *Phys. Rev.* D **48** 137
[40] Enchenard B (on behalf of the L3 Collaboration) 2004 *Eur. Phys. J.* C **33** 566